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Strategies for Improving Mathematics Problem-Solving Skills in Elementary Aged Students with Learning Disabilities

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Strategies for Improving Mathematics Problem-Solving Skills in Elementary Aged Students with Learning Disabilities

By

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Strategies for Improving Mathematics Problem-Solving Skills in Elementary Aged Students with Learning Disabilities

By

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The purpose of this synthesis was to find research-based strategies that would be effective in helping elementary aged students with mathematics difficulties improve their problem-solving skills. In this synthesis, articles were included based on their relevance and the following is a summary of their findings.

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Introduction

Mathematics skills can be broken into two primary areas: procedural skills (the ability to perform calculations), and conceptual skills (the ability to think abstractly and solve problems) (Seethaler & Fuchs, 2010). Researchers have found that students with disabilities struggle with conceptual skills in particular because they lack the necessary calculation skills, the ability to comprehend concepts within word problems, as well as the ability to effectively use previously learned strategies (Andersson, 2007; Fung, Swanson, & Orosco, 2014; Garrett, Mazzocco, & Baker, 2006; Rosenzweig, Krawec, & Montague, 2011).

Within the area of conceptual skills, problem solving can itself be broken into two areas as well: problem comprehension (a student's ability to verbally express what was stated in a word problem (Mayer, 1985); as well as a student's ability to create a number sentence from the linguistic components of a word problem (Lewis & Mayer, 1987; Mayer, 1985; Mayer & Hegarty, 1996)), and problem solution (Hegerty, Mayer, & Monk, 1995; Mayer & Hegarty, 1996).

Being able to solve word problems is an important skill because it most closely resembles the ability to solve mathematical problems in the real world (Swanson, 2014). But, calculation skills and problem solving skills cannot be taught in the same fashion because they require different cognitive processes (Fuchs, Fuchs, Stuebing, Fletcher, Hamlett, & Lambert, 2008). Being able to identify effective strategies for teaching students with mathematics difficulties (MD) is the basis for this synthesis, and the following question guided the search for relevant research:

- What strategies can teachers use to improve the mathematical problem solving skills in elementary-aged students that have, or are at risk of having, mathematics difficulties or learning disabilities (MLD)?

Methods

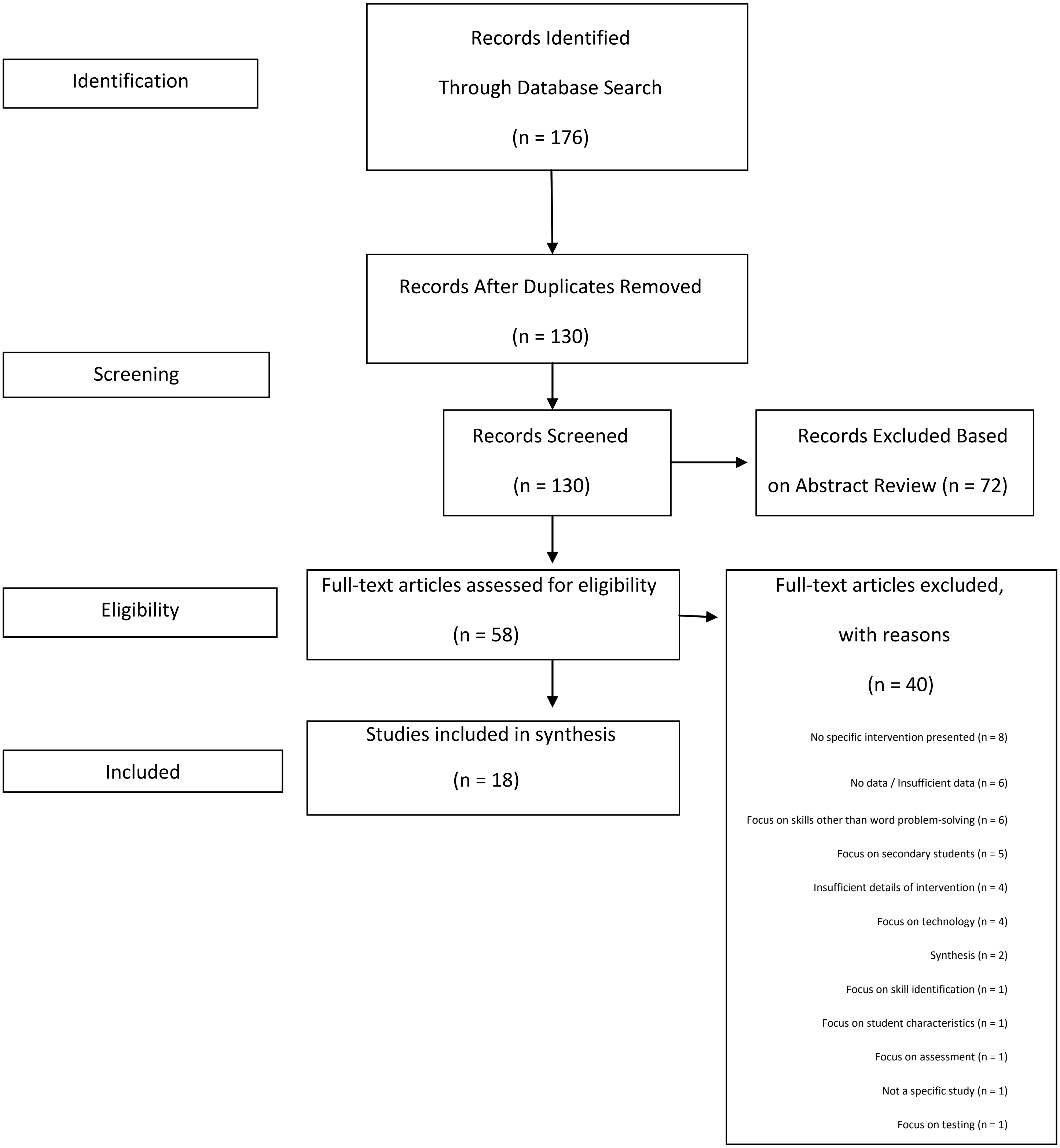
This literature review began by going on to the University of Texas at Austin's library website (www.lib.utexas.edu), and searching for the EBSCO database. After the search, I followed the link for the Education Source database. In addition to Education Source, both the ERIC and PsycINFO databases were used to search for relevant articles.

Within the databases I limited my search to peer-reviewed articles that were written in English, whose participants were school-aged (6 to 12-years-old) students in grades 1 through 6, whose full text was available in the databases, and were published in or after 1998. Inclusion criteria also consisted of articles that focused on mathematical problem-solving skills, and only included students in elementary school that had, or were at risk of having, a learning disability. Articles were excluded if they included secondary students, strategies that could not be implemented by a teacher, or focused on an area other than mathematical problem solving.

After applying all of the limiting factors, the search resulted in 176 articles to be reviewed. Within the search there were 46 duplicates, which left 130 articles to be screened. Upon review of their abstracts I was able to eliminate 72 articles for various reasons (e.g., participants were not in elementary school, strategies could not be implemented by a teacher, etc.). This left me with 58 articles to review based on their full text.

Of the 58 articles that I reviewed in the final stage of article selection, 40 were eliminated for various reasons, which are outlined in the following PRISMA (Figure 1):

**Figure 1*



This PRISMA shows the process that I took to find relevant articles for this synthesis.

Literature Review

During the process of reviewing literature, I found that strategies could be placed into one of the following categories: cognitive strategies, ad hoc tutoring, or strategies for English Language Learners. In this section I will discuss the types of strategies found in the literature.

Cognitive Strategies

Research has determined that when a student is trying to solve a word problem there are several things that need to happen for a student to be able to do that: they need to be able to (1) comprehend the meaning of the text, (2) identify the type of word problem, (3) visually represent what they have read, (4) solve the problem, (5) check the answer, and (6) adequately convey the answer (Kingsdorf, & Krawec, 2016). Researchers have learned that students struggle more with understanding, visually representing, and coming up with a plan on how to solve a problem than they do with the actual calculation skills involved (Montague & Applegate, 1993).

Within the area of mathematical problem solving, researchers have outlined cognitive strategy instruction (CSI), which addresses both a student's ability to identify what they are being asked to do as well as their ability to monitor their own cognitive processes while completing a task (Krawec & Montague, 2012; Montague, 2007), as a way to teach students how to visualize what is going on in a word problem and improve their metacognitive skills so that solving mathematical word problems becomes easier (Montague & Deitz, 2009). Previous research has shown that students, particularly those with mathematics difficulties (MD), who struggle to choose the appropriate strategy when attempting to solve word problems can benefit from cognitive strategy instruction (Coughlin & Montague, 2011). In this section I discuss research related to word problems with a focus on cognitive strategies. I describe each study and provide implications for practice. I will begin by synthesizing two articles that researched cognitive strategy instruction in a broad sense, and then move on to articles that focus on more specific areas of cognitive strategy instruction: three articles studied the effects of schema based instruction, two studied the use of a conceptual model-based problem-solving strategy, three studied the use of generative strategies, one studied the use of a bar model strategy, three studied the effect of students being able to identify both relevant and irrelevant information, and one article studied the effects of self-explanation.

With improving both metacognitive skills and performance on cognitive tasks in mind, Zhu (2015) conducted a study, in which she selected 150 fourth-grade students (75 in the experimental group; 75 in the control group) in China to participate, that sought to determine the effectiveness of using cognitive strategy instruction to improve word-problem solving skills (Zhu, 2015). The intervention consisted of a cognitive strategy used in conjunction with three metacognitive strategies (Zhu, 2015). The cognitive strategy included seven steps: (1) read the problem, (2) restate the problem, (3) use a graphic organizer to represent the problem, (4) make a plan on how to solve the problem, (5) estimate the answer to the problem, (6) solve the problem, and (7) check your answer to the problem (Montague, 1992). Zhu also used the following metacognitive strategies: (1) say (the participants would tell themselves what to do), (2) ask (the participants would ask questions of themselves), and (3) check (the participants would continually check their own work) (Zhu, 2015).

Results from Zhu's (2015) study showed that participants in the intervention group who were considered to be performing on an average level, as well as those with MD, improved in their word-problem solving accuracy (Zhu, 2015). This study is important because it was not only successful, but it also took place in China. So, even though many of the other studies that I will review in this synthesis occurred in the United States, Zhu's work shows that strategies that are successful in the United States have the potential to be successful in other parts of the world as well.

In 2016 Kingsdorf and Krawec completed a study, which was able to extend the research of Zhu (2015), that had the goal of learning what the effect of direct instruction with numerous examples, along with the use of a self-monitoring checklist, would be on the word-problem performance of 10 third-grade participants to accurately paraphrase, visually represent, and solve mathematical word problems (Kingsdorf & Krawec, 2016). These are all important components of cognitive strategy instruction. The first lesson of the intervention consisted of instructors directly teaching the participants how to paraphrase a word problem through the use of modeling and a think-aloud (Kingsdorf & Krawec, 2016). Then from the second lesson onward, until at least eight students met mastery criterion (i.e., 7/8 students were able to accurately solve problems across two consecutive sessions), the instructors modeled how to paraphrase a new word problem at the beginning of each lesson, which was then followed by an assessment during which participants were allowed to use a paraphrasing self-monitoring checklist (Kingsdorf & Krawec, 2016). After enough participants had sufficiently mastered paraphrasing, the visual representation intervention began, which took the same format as the paraphrasing intervention (Kingsdorf & Krawec, 2016).

Overall, Kingsdorf and Krawec (2016) found that the intervention improved participants' abilities to paraphrase, visually represent, and accurately solve word problems. Because there were multiple components to this study (e.g., a paraphrasing intervention, and a visual representation intervention), the authors were unable to definitively say which portion of the intervention was the most beneficial for word-problem solving (Kingsdorf & Krawec, 2016).

The research by Zhu (2015) and Kingsdorf and Krawec (2016) studied the use of cognitive strategy instruction in its broadest sense. In the following sections, I describe strategies that target more specific areas of cognition. All of the strategies, though, still fall under the umbrella term of CSI.

Schema-based instruction. Specifically designed to improve the problem solving abilities of students with MD, the goal of schema-based instruction is to teach students how to identify the root structure of word problems (Morin, Watson, Hester, & Raver, 2017).

Word problems are structured in such a way that students are required to not only identify the elements of the problem, but to use those elements to create a number sentence that can then be solved (Fuchs, Powell, Seethaler, Cirino, Fletcher, Fuchs, Hamlett, & Zumeta, 2009). Because of these extra steps, studies have suggested that calculation and word problem solving require somewhat different cognitive abilities (Fuchs, Fuchs, Compton, Powell, Seethaler, Capizzi, 2006).

In her 2009 study, Fuchs et al. researched the effect that two separate interventions had on two areas of difficulty: number combinations and word problems. A total of 133 participants, all third-graders, were randomly assigned to one of three conditions: number combinations tutoring, word-problem tutoring, or control group. Each condition included students that struggled solely with MD, as well as those that had MD in conjunction with reading difficulties.

Because the focus of this synthesis is word-problem solving, I focus on the components and effects of the word-problem tutoring group.

The word-problem tutoring program implemented by Fuchs et al. (2009) was based on schema-based instruction, where each unit of the program focused on a different type of problem (e.g., total, difference, or change). Within each unit of the program, participants were instructed on the use of a three-step strategy, which guided them through the reading of the problem, an underlining of the question, and a naming of the problem type. The participants were then guided through how to label important information, which differed depending on the type of problem (Fuchs et al., 2009). Overall, the tutoring lessons in each unit had four components: a warm-up using flash cards of basic facts, conceptual and strategic instruction, a sorting activity using word-problem flash cards, and a paper and pencil review.

Results from Fuchs et al. (2009) showed that both types of intervention proved to be beneficial in terms of number combinations fluency. This is important because previous research has shown that strong number combinations fluency can positively affect both procedural computation and word-problem solving skills (Fuchs et al., 2009). With respect to procedural computation skills, while both intervention groups showed improvement, the effect size for the word-problem tutoring group (the group whose intervention focused on problem solving skills) was nearly twice that of the number combinations tutoring group (the group whose intervention focused on calculation skills) at 0.53 and 0.27, respectively (Fuchs et al., 2009). Importantly, participants that were in the word-problem tutoring group, but not the number combinations group, also showed improvement in both algebraic skills and all measures of word-problem performance (Fuchs et al., 2009).

It is important to note that Fuchs et al.'s (2009) hypothesis that improving a participant's number combinations skills would lead to improved word-problem scores was not supported. The authors learned that the primary causes of the students' struggles were the students' ability to "comprehend their relations among the numbers embedded in the narratives or to process the language in those stories adequately" (Fuchs et al., 2009, p. 572). For practical purposes, this means that word-problem instruction has to be focused on comprehension and language abilities rather than calculation skills alone, which is one of the central ideas of cognitive strategy instruction (Montague & Deitz, 2009).

Researchers in the past have also determined that, besides having to identify and create a number sentence using the various elements of word problems (Fuchs et al., 2009), students need to be able to choose an appropriate strategy to solve the given problem, as well as understand the language of the problem (Swanson & Beebe-Frankenberger, 2004). This again goes back to the goal of improving metacognitive skills seen in cognitive strategy instruction (Krawec & Montague, 2012; Montague, 2007).

The idea of having to choose appropriate strategies and having to understand a problem's language (Swanson & Beebe-Frankenberger, 2004), paired with previous findings that learning improves when students have the opportunity to converse with each other about their own mathematical thinking in a small group setting (Van Luit, & Naglieri, 1999), guided the research question in Jitendra and her colleagues' (2013) study (Jitendra, Rodriguez, Kanive, Huang, Church, Corroy, & Zaslofsky, 2013). Jitendra et al. wanted to determine the efficacy of using schema-based instruction in a small group setting as compared with a standards-based curriculum. The theoretical basis of schema-based instruction is that "multiple elements of information are grouped into and conceptualized as a single schema, [and that] recognizing a problem's schema reduces the working memory load during cognitive processing" (Jitendra, et al., 2013, p. 22).

In the study of 136 third-grade participants, 72 received the schema-based instruction tutoring and 64 received instruction within the standards-based curriculum (Jitendra et al., 2013). During the 30 min of supplemental tutoring that participants in the schema-based group received for five days per week for 12 weeks, students worked on 21 different lessons that covered five units (i.e., change, group, compare, review, and two-step problems). The first three units included five lessons, with the first lesson of each unit consisting of the participants using stories that had no unknown quantities to practice identifying the problem's schema (Jitendra et al., 2013). During this lesson, participants were also introduced to diagrams and problem-solving checklists to help students with the word-problem process. Lessons two, three, and four consisted of direct instruction in the use of the given diagrams and problem-solving checklists. In the final lesson of each of the first three units, students created their own diagrams, instead of using the ones given, to help solve the problems (Jitendra et al., 2013). The fourth unit of the program was a review of the first three units, which took place over three lessons (one for each problem type). Finally, the fifth unit introduced the participants to two-step problems over the course of three lessons, and then reviewed both one- and two-step problems.

Jitendra et al. (2013) found that, among the participants that scored higher on the pretest, students that received the schema-based intervention performed better on the posttest than those that received the standards-based curriculum that was provided by the school (Jitendra et al., 2013). But, Jitendra et al. also found that the participants that had lower pretest scores benefited more from the standards based curriculum, at least as shown by posttest scores. The authors speculated that this difference might have been caused by some students' lack of basic skills in mathematics (Jitendra et al., 2013). The authors suggested that because word-problem solving skills require higher order thinking skills, the participants that had not yet become proficient in the basic mathematical skills (i.e., the participants that scored lower on the pretest) did not benefit as much from the schema-based intervention. These findings indicate that using schema-based instruction in a small-group setting can be beneficial for students at risk for MD when it comes to improving word-problem solving skills (particularly for those that have relatively high problem solving scores, as compared to other students with mathematics difficulties; Jitendra et al., 2013).

Another challenge in trying to improve performance on cognitive tasks in students with MD as they relate to word-problem solving, in addition to supporting their understanding of a problem's language (Swanson & Beebe-Frankenberger, 2004) and elements (Fuchs et al., 2009), as well as their ability to choose an appropriate problem solving strategy (Swanson, & Beebe-Frankenberger, 2004), is that traditionally, the learning of mathematics has been thought of as having a vertical progression, wherein one is able to naturally progress to more difficult concepts once prerequisite skills have been mastered (Gagne, 1968; Resnick & Resnick, 1992). The problem with this vertical concept is that when learning the basic skills, research has found that students struggle to apply those concepts in multiple contexts (Boaler, 1993). This is why the way that mathematics is being taught in American schools has shifted from trying to help students transfer skills vertically to teaching how to transfer skills laterally, or generalize strategies (e.g., being able to recognize that a strategy can be used with various problems; Fuchs, Fuchs, Craddock, Hollenbeck, Hamlett, & Schatschneider, 2008).

The ability to recognize that various problems can be of the same type, and that the same strategy can be used for all of them, has been referred to by others as the development of schemas (Brown, Campione, Webber, & McGilly, 1992; Gick & Holyoak, 1983). When someone refers to a schema, they are referring to a problem's type (e.g., change, compare, etc.; Chi, Feltovich, & Glaser, 1981; Gick & Holyoak, 1983; Mayer, 1992; Quilici & Mayer, 1996). Based on all of this information, Fuchs and colleagues conducted a study in 2008 to determine what effect both small-group tutoring and whole-class instruction that utilized schema-based instruction would have on the learning of students that were at-risk for MD when compared to same-aged students who were not at risk (Fuchs, Fuchs, Craddock, et al., 2008).

Over a 4-year period, 1,141 third-grade students, across 120 classrooms, participated in the study (Fuchs, Fuchs, Craddock, et al., 2008). Of the 120 classrooms that participated, 80 of the classrooms were randomly assigned to receive the whole-class schema-based instruction. During whole-class schema-based instruction students received instruction that covered four units, with each unit lasting three weeks. Each unit consisted of six lessons, with the first four lessons having the same number sentences but varying stories behind the numbers. In these first four lessons, the research assistants taught the participants the structure (i.e., schema) of each type of problem and also used posters that outlined the steps to solving the problems to guide the participants through the problem-solving process. The fifth and sixth lessons not only changed the stories behind the problems, but superficial features as well. The superficial features emphasized the fact that problems can be of the same type even if one or more uses unfamiliar vocabulary, poses additional questions, incorporates irrelevant information, or combines problem types. The researchers provided participants with a poster that outlined these features as well.

For the tutoring groups, two-thirds of participants who were considered at-risk for MD were assigned to the schema-based tutoring group, with the remaining one-third assigned to a control group (Fuchs, Fuchs, Craddock, et al., 2008). The schema-based tutoring content was similar to that used during the whole-class sessions. The tutoring sessions differed, though, in that the sessions focused on the trickiest concepts from schema-based instruction provided at the whole-class level. Small-group tutoring also used more manipulatives, provided more support to participants through the use of scaffolding, and used self-regulated learning strategies with tangible reinforcements (Fuchs, Fuchs, Craddock, et al., 2008).

Fuchs, Fuchs, Craddock, et al. (2008) determined that tutoring did benefit students who were at-risk for MD, and that effects were much more pronounced when small-group tutoring was combined with whole-class instruction. This finding validated previous findings that multi-tiered systems of support, or the three tiers of response to intervention, are beneficial to students (Fuchs, Fuchs, Craddock, et al., 2008). The key, though, is that instruction at all tiers (whole-class and small-group) needs to be research-based and of the highest quality.

This research relating to schema-based instruction demonstrates that students with MD benefit from explicit instruction related to word-problem schemas. Students also benefit from cognitive strategies embedded within schema-based instruction, especially learning how to work through and solve word-problems step-by-step. This knowledge transfers to word problems that are unfamiliar to students and allows for transfer.

Conceptual model-based problem-solving (COMPS). One of the reasons for a decline in mathematics skills as students get older (National Mathematics Advisory Panel, 2008) is because around the fourth grade, there is a shift to a more abstract way of thinking and the use of symbols becomes more prevalent (Cai, Lew, Morris, Moyer, Ng, & Schmittau, 2004). Students with learning disabilities are at a particular disadvantage during this transition because of their struggle with cognitive processing skills (Xin & Zhang, 2009). This is where conceptual model-based problem-solving comes in, because it teaches students how to model a problem's structure in a way that can be transferred and used to solve other types of problems (Xin, Zhang, et al., 2011). This strategy is similar to schema-based instruction, with the difference being that this strategy is not limited to addition and subtraction problems (Xin, Zhang, et al., 2011).

In a 2011 study, Xin et al. implemented a conceptual model-based problem-solving approach, and compared it against a general heuristic instructional approach to determine whether it could improve students' ability to solve word-problems (Xin et al., 2011).

The primary instruction in the COMPS approach occurred using a graphic organizer that helped participants visualize how quantities from the problem(s) related to each other, and then utilized direct problem-solving instruction (Xin et al., 2011). The model representation portion of the lesson consisted of participants being instructed to identify the type of problem they were working with, and then to transfer the information from the problem onto the diagram provided. An important aspect of this portion of the lesson was that when students were just beginning to break apart problems in this way, there were not any unknown quantities in the problem. Providing all quantities in a word problem provided students the opportunity to visualize how the quantities relate to each other.

After instruction in model representation, direct instruction took place in problem solving (Xin et al., 2011). This portion of the lesson differed from the model representation portion in that students used a four-step checklist, called DOTS, to guide them through solving problems with an unknown quantity. The first step of the DOTS checklist is to "detect the problem type based on the problem structure they learned during model representation instruction" (Xin et al., 2011, p. 386). Next, the students organized information from the problem using the diagrams from the model representation instruction portion. Then, students would transform the information from the diagram into an equation by removing, or labeling the conceptual model diagram so students were left with a mathematics equation. Finally, the student would solve the equation by determining the value of the unknown quantity.

The general-heuristic approach was guided primarily by the five-step checklist SOLVE (Search-Organize-Look-Visualize-Evaluate; Xin et al., 2011). The difference between this model and the COMPS model is that this model provides the flexibility to choose from various strategies.

After the intervention phase was completed, when comparing pre- and posttest scores, it was determined that the COMPS group improved significantly more than the general heuristic group with respect to criterion word problem-solving skills and on pre-algebra model expression skills (Xin et al., 2011). It is important to note that there was not a significant difference in pretest and posttest scores for far transfer skills. The insignificant difference in scores for the far transfer measure is thought to be attributable in part to the fact that the particular measure that was used only contained a small number of word problems that related to the types of problems that were taught in the study. So this intervention proved to be successful only when measuring performance on skills taught, which might indicate that the intervention would be ineffective for teaching transfer skills.

Xin's (2011) study was similar to a study conducted in 2009, where she and colleagues outlined the conceptual model-based problem-solving strategy as a way to improving problem solving by focusing on the relationships between quantities found in word problems (Xin, Wiles, & Lin, 2008).

Xin and Zhang (2009) implemented this strategy with three participants who were in either the fourth- or fifth-grade. During the intervention phase of the study, participants received instruction on how to solve word problems using a checklist named DOTS (Detect-Organize-Transform-Solve; Xin & Zhang, 2009). The interventionists first taught the participants to identify (or detect) the type of problem, then to use the given graphic organizer to organize the information from the problem, after which the participants had to transform the information from the graphic organizer into a number sentence, and finally solve the number sentence (Xin & Zhang, 2009).

Results from Xin and Zhang (2009) showed that all three participants were able to improve their word problem solving skills from pretest to posttest. The authors noted that a benefit of using this strategy is that students are not required to remember rules to solve problems because the graphic organizers that are provided help students to visualize the structure of the problem they are working with and what the quantities the students work with mean (Xin & Zhang, 2009). Further research to determine the effect this strategy would have on the problem-solving accuracy of students with working memory capacity issues may be beneficial.

Generative strategies (paraphrasing). While schema-based instruction and the conceptual model-based problem-solving strategy focus on students' ability to identify types of word problems to improve comprehension, generative strategies target students' ability to summarize the content of word problems with the same goal of improving comprehension (Moran, Swanson, et al., 2014).

Even though most research conducted on problem-solving in mathematics has focused on schemas, Moran, Swanson, Gerber, and Fung (2014) completed a study that focused more on how the reading and writing intervention of paraphrasing could help students improve with regards to the cognitive processing area of problem solving (reading comprehension in particular). More specifically, Moran's study dealt with what is called problem translation, or one's ability to exhibit their understanding of a word problem by restating both the relevant and irrelevant information of a problem (Bull & Epsy, 2006; Cain, 2006). But, where this study differs from problem translation is that participants were being tasked with writing out the various components of the problems they were given (Moran et al., 2014). The writing process was an important aspect of Moran's study because of its effectiveness in helping students recall information (Bakken, Mastropieri, & Scruggs, 1997; Gajria & Salvia, 1992; Nelson, Smith, & Dodd, 1992).

When presented with word problems, the 72 third graders that participated in Moran et. al.'s (2014) study were assigned to one of three paraphrasing conditions: the restate proposition condition, the relevant proposition condition, and the complete proposition condition. Participants were instructed to rewrite, in their own words, the following items for each condition (Moran et al., 2014):

- Restate Proposition Condition – the question in the word problem
- Relevant Proposition Condition – the question, as well as relevant information

- Complete Proposition Condition – the question, relevant information, and irrelevant information

During the intervention phase, the 20 lessons consisted of five problems and were divided into an I do, we do, you do (i.e., explicit instruction) model (Moran et al., 2014). The tutor modeled how to do the first problem of the lesson, guided the students through the second problem of the lesson, and had the students independently complete the final three problems of the lesson. One final aspect of the intervention was that the first five lessons had the sole purpose of providing the participants time to practice identifying the components of the word problems they were tasked with finding, without requiring them to restate the components in their own words (Moran et al., 2014).

After the completion of the intervention, Moran et al. (2014) determined that paraphrasing was an effective strategy for students in both the complete and relevant conditions with respect to problem-solving accuracy. Participants in both conditions outperformed their peers that received the restate condition, as well as peers in the control group (Moran, Swanson et al., 2014). Therefore, to improve comprehension of word problems, students must receive at least some instruction on the ability to identify the various components of a problem.

Swanson, Moran, Lussier, and Fung (2014) conducted a second study on generative strategy training to determine whether the positive effects seen in the last study could or would be influenced by the working memory capacity of participants. The goal for this study was not only to determine the effect that generative strategy training has on word problem-solving accuracy, but to determine to what degree a participant's low working memory may affect success with the strategy (Swanson, Moran, et al., 2014).

Swanson, Moran, et al. (2014) had the same basic setup and procedure as the previous study (Moran et al., 2014). The 82 participants of this study, all from the third grade, were assigned to four different groups: the restate condition, the relevant condition, the complete condition, and the control group. Each of the 20 lessons followed an explicit instruction structure (Swanson, Moran, et al., 2014). The primary difference between this study and the previous one is that it utilized a test which aided in the measurement of the participants' working memory (Swanson, Moran, et al., 2014).

Like Moran et al. (2014), Swanson, Moran, et al. (2014) determined that generative strategy training was an effective intervention when trying to improve accuracy with respect to word problem-solving, but only for participants that had a relatively high working-memory capacity (Swanson, Moran, et al., 2014). More specifically, Swanson, Moran, et al. noted in this study that it was the participants in the complete condition (versus the restate and relevant conditions) that had the advantage over the control group. So, when implementing a strategy in the classrooms, teachers should keep in mind that its effectiveness may depend on the working memory of their students because some strategies are more taxing, cognitively, than others (Swanson, Moran, et al., 2014).

Both of these 2014 studies by Swanson, Moran, et al. and Moran et al. were able to replicate a similar study, conducted by Swanson, Moran, Bocian, Lussier, and Zheng in 2012. The 2012 study had the same basic format, and looked at the effect that generative strategies had on students' ability to accurately solve mathematical word problems. The 2012 study came to the same conclusion: that participants in both the relevant and complete condition groups improved their scores from pretest to posttest with respect to word problem solving accuracy (Swanson, et al., 2012).

The findings by Swanson, Moran, et al. and Moran et al.'s studies that generative strategies positively affect students' problem solving skills is important because not only is the ability to paraphrase a component of cognitive strategy instruction, but the ability to summarize is an important aspect of reading comprehension in general (Montague, 1992; National Reading Panel, 2000). More specifically, with respect to cognitive strategies, paraphrasing has been found to be beneficial for students' working memory capacity (Bakken, Mastropieri, & Scruggs, 1997; Gajria & Salvia, 1992; Holmes, Gathercole, & Dunning, 2009; Nelson, Smith, & Dodd, 1992; Turley-Ames & Whitfield, 2003).

Bar model. Research has found that another reason why students with MD struggle with word problems in mathematics is that students lack either the necessary computation skills, an understanding of mathematical concepts, or the ability to successfully use strategies that have been taught (Andersson, 2007). To help students that struggle in these areas, Morin devised a strategy based on cognitive strategy instruction, which uses explicit instruction to improve students' metacognitive skills (Krawec & Montague, 2012), as well as schema-based instruction, which helps in improving students' understanding of word problem structure (Jitendra et al., 2013).

Morin, Watson et al.'s (2017) study consisted of guiding six participants, all in the third grade, through the use of the bar model as a way to help them visualize the types of word-problem schema that have been described in previous schema-based literature (change, group, compare, vary, equal-group, and part-whole; Jitendra, DiPipi, & Perron-Jones, 2002; Jitendra, Griffin, Deatline-Buchman, & Sczesniak, 2007; Jitendra & Kameenui, 1993). To help facilitate the use of the bar model, Morin provided participants with a list of seven steps that were to be used when solving word problems (Morin, Watson, et al., 2017). The first step on the list was for the student to read the problem in its entirety. Morin was able to implement cognitive strategy instruction into his intervention through the other six steps on the list, which integrated common cognitive strategies into the process of using the bar model (Morin, Watson, Hester, & Raver, 2017). At the end of the seven steps, students, or participants in this case, created a visual representation of the given word problem that was essentially a long rectangle (or bar) showing both the known and unknown quantities of the problem (Forsten, 2010).

Based on visual analysis of data, Morin, Watson et al. (2017) were able to confirm, at least with the participants in this study, that the use of the bar model was a success in improving the participants' ability to use cognitive strategies to solve word problems. The students went from using virtually no cognitive strategies to solve problems during baseline, to consistently using the strategies during the maintenance phase. Not only did the bar model help in the participants' ability to use cognitive strategies, but it also helped to improve their accuracy in solving word problems. Based on these findings, it is important to explicitly teach cognitive strategy skills, as well as ways to visually represent word problems to students with learning disabilities.

Identifying relevant and irrelevant information. Researchers have shown that the use of cognitive strategies with students with MD is beneficial because it helps to reduce how much the cognitive processes of students are taxed (Swanson, Orosco, & Lussier, 2014). Therefore, it is important to keep in mind, when choosing a strategy to implement, how much working memory a student has available to facilitate the use of the given strategy (Swanson, Orosco, et al., 2014).

In a 2014 study, Swanson, Orosco, et al. set out to determine if, and by how much, the use of various cognitive strategies that focused on identifying relevant information in word problems would improve the word problem solving accuracy of participants identified as having a math disability. Swanson also wanted to determine if certain strategies were more effective than other strategies. This study was different than other similar studies because the intervention helped participants find relevant information when students were presented with irrelevant information as well (Swanson, Orosco, et al., 2014). Helping participants discriminate between relevant and irrelevant information was an important aspect of this study because previous research has shown that this is a difficult skill to learn, especially for elementary-aged students (Cook & Rieser, 2005).

Within the study, 193 third-grade students were randomly assigned to: verbal strategies, visual-spatial strategies, a combination of both verbal and visual-spatial strategies, and a control group (i.e., materials-only) (Swanson, Orosco, et al., 2014). The verbal strategy consisted of the identification and labeling of four items: the question, numbers, key words, and irrelevant information. The participants were then instructed to find the operation that needed to be performed, and to solve the problem. With the visual-spatial strategy, instruction consisted of using two graphic organizers to teach participants how parts make up a whole and how quantities can be compared (Swanson, Orosco, et al., 2014). Participants assigned to the verbal and visual-spatial condition were instructed using the same steps from the verbal strategy, but with the additional step of filling in a graphic organizer with the information that they found and identifying the unknown quantities (e.g., part or whole). Finally, within the materials-only group, participants used the same curriculum materials as the participants in the other groups, but were not exposed to explicit instruction of a particular strategy. The purpose of the materials-only group was to determine whether the increase in irrelevant information itself could improve problem solving accuracy.

Results from Swanson, Orosco, et al. (2014) indicated that, with respect to problem solving accuracy, participants in the visual-spatial and materials-only strategies performed better than those in the control group (Swanson, Orosco, et al., 2014). The authors suggest that the materials-only condition might have had a positive effect on posttest scores because of the participants' struggles with working memory capacity, and the fact that the materials-only condition did not have a particular strategy that the participants had to remember. The authors also speculated that the verbal and visual-spatial condition was less taxing on the participants' working memory, which is what led to the observed improvement. This indicates that, if students struggle with working memory, teachers should try to choose a strategy that is the least taxing to those resources.

But, how much of an effect does a student's working memory capacity have on the ability to use certain strategies? This was the purpose of Swanson's 2014 study: to determine how much of an effect a student's working memory capacity has on their ability to successfully use cognitive strategies (Swanson, 2014).

In his study, Swanson randomly assigned 147 third graders to one of four conditions: verbal only, verbal and visual, visual only, and control. The intervention phase of the study lasted for eight weeks, and participants received instruction in 20 lessons (Swanson, 2014). Each lesson consisted of a warm-up, and then followed an I do, we do, you do (i.e., explicit instruction) model wherein the instructors first modeled how to use their respective strategy, guided the participants through the use of the strategy, and finally had the participants practice the strategy independently.

The participants in the verbal only strategy group were taught to underline the problem's question, circle the numbers, mark key words with a square, cross off irrelevant information, identify which operations need to be perform, and to finally solve the problem. The participants in the visual only strategy group were taught to use two graphic organizers to represent information from the word problems (Swanson, 2014). One of the graphic organizers represented how parts in a whole and how quantities were compared (Swanson, 2014). Finally, the participants in the verbal and visual strategy group were taught to follow the same steps as the participants in the verbal only group, with the added step of using a graphic organizer to represent the known quantities from their given problem and having to solve for the unknown quantity (Swanson, 2014).

Results from the Swanson (2014) study indicated that students whose working memory scores were relatively high benefited from the use of the both the verbal only and visual only strategy, with the visual only strategy proving to be the most beneficial. For participants whose working memory scores were relatively low, there was no significant benefit for any of the conditions (Swanson, 2014). So, the author was able to affirm that a student's working memory does have an impact on whether or not they are able to successfully use cognitive strategies (Swanson, 2014).

Both of Swanson's studies in 2014 were similar to a study completed in 2013. In that study, Swanson, Lussier, and Orosco (2013) conducted a study to investigate whether or not cognitive strategies could improve the accuracy of students with MD in word-problem abilities when compared to a control group. While reviewing previous literature, Swanson found that both visual-spatial, (Kolloffel, Eysink, de Jong, & Wilhelm, 2009; van Garderen, 2007 as cited by Swanson, 2013) and general-heuristic (Montague, 2008; Montague, Warger, & Morgan, 2000 as cited by Swanson, 2013) strategies had proven to be especially effective in improving mathematical skills (Baker, Gersten, & Lee 2002; Gersten, Chard, Jayanthi, Baker, Morphy, & Flojo, 2009).

As the name implies, visual-spatial strategies teach students to use a graphic organizer to represent information found in word problems (Powell, 2011; Van Garderen & Montague, 2003). General-heuristic strategies on the other hand task students with marking both relevant (Brissiaud & Sander, 2010; Hayes, Waterman, & Robinson, 1977) and irrelevant (Low & Over, 1989) information within the text of a word problem itself.

In the 2013 study, Swanson et al. randomly assigned 120 third-grade students to one of four treatment conditions: general-heuristic, general-heuristic and visual-schematic, visual-schematic, or control. Each treatment condition lasted for eight weeks and consisted of 20 lessons. Within each lesson the participants began with a warm-up and were then instructed on their respective intervention using the I do, we do, you do (i.e., explicit instruction) model wherein the instructor would first teach the strategy, then guide them through the use of the strategy, and finally have the participants practice the strategy independently.

During the general-heuristic strategy participants were instructed to underline the question in the problem, circle the sentences that included numbers, mark the key words from the problem with a square, cross out any irrelevant information, identify which operation(s) needed to be performed, and to finally solve the problem (Swanson et al., 2013). During the visual-schematic strategy, participants utilized two graphic organizers: one helped the participants visualize "how parts made up a whole and the second represented how quantities are compared" (Swanson et al., 2013, p. 173). Finally, the participants in the general-heuristic and visual-schematic group solved problems using the steps of the general-heuristic strategy in conjunction with the use of a graphic organizer where they filled in the quantities that were known, and had to solve for the unknown quantities.

Results from this study demonstrated that participants in the visual-schematic strategy outperformed participants in the control group on assessments related to problem-solving accuracy, calculation accuracy, and problem-solving concepts (Swanson et al., 2013). The authors suggested that a possible reason as to why the visual-schematic strategy was more beneficial than the other strategies was that it was less taxing to students' working memory capacity than the general-heuristic strategy.

Self-explanation. So far, the studies I have reviewed have looked at strategies for helping students accurately solve mathematics word problems. But, an important aspect of word problem solving is a student's ability to effectively communicate not only their answer to the problem, but also how they arrived at that answer (Liu & Xin, 2017). This skill forms the foundation of self-explanation, which is the idea that a dialogue between students can help to reinforce what students already know, and also provide guidance for those that are struggling

(Kotsopoulos, 2010). Though there has not been an abundant amount of research on the effect that self-explanation has on students with MD, there has been research that has shown that typically-developing students who are able to successfully explain themselves typically show improved performance on various cognitive tasks, which is one of the goals of cognitive strategy instruction (Aleven & Koedinger, 2002). Because of the effects that have been seen with students in general education, Liu and Xin (2017) implemented a scaffolding technique that targeted conversational skills with three fourth-grade participants that were all identified as having a learning disability (LD), with the goal of finding the effect that self-explanation has on students with disabilities.

Liu and Xin's (2017) strategy consisted of four levels of prompts, meant to scaffold the participants' self-explanation skills: general request, request for revision, request for clarification, and direct teaching. When using the general request the interventionist would ask the participant, "Do you want to try again to make it better?" if either the participants explanation was not exact, or the participant's answer was not correct (Liu & Xin, 2017, p. 138). If a participant's explanation was deemed incorrect after the general request, then the interventionist would request a revision, wherein explanations of the two quantities from the given problem would be provided before the participant is asked if they want to revise their explanation. After the request for revision prompt, if a participant's explanation was considered to be basically correct, but still too general, the participants were asked why they did what they did to solve the problem. This prompt was considered the request for clarification. Finally, if the participant's response was still not exact enough, the interventionist would move into direct teaching, using story grammar question cards, to teach the participants how to recognize the components of the problems being solved, represent those components on the graphic organizer that took the form of a number sentence, and solve the problem using the number sentence (Liu & Xin, 2017).

Results from Liu and Xin (2017) showed that when provided the appropriate scaffolding, the participants were able to improve in their self-explanation skills, when compared to the baseline phase. As with their self-explanation skills, the participants' problem-solving skills also improved, and at a more rapid pace (Liu & Xin, 2017). As far as what this means for the classroom, improving a student's ability to verbally communicate how the student arrived at an answer does have a positive effect on their ability to solve problems (Liu & Xin, 2017).

Ad Hoc Tutoring

Strategies to help students with, or who are at risk of having, learning disabilities improve their problem solving skills are not limited to those based on cognitive strategy instruction. When discussing how to improve the mathematics abilities of students with MD it is generally accepted that students need instruction beyond that that can be provided by the general education teacher (Woodward, Monroe, & Baxter, 2001). One strategy to provide more intensive instruction to students with MD is through the use of ad hoc tutoring, which differs from the standard curriculum in that it focuses more on the day-to-day needs of individual students (Good, Mulryan, & McCaslin, 1992; Woodward, Baxter, Olson, & Kline, 1996). When implementing ad hoc tutoring, general education teachers provide whole-class instruction while tutors or paraprofessionals provide additional instruction to students in the specific concepts that they struggle with (Woodward et al., 2001).

In a 2001 study, Woodward et al. assigned 182 fourth-grade students to either an intervention group that received ad hoc tutoring and participated in whole-class assessments, or to a control group (Woodward et al., 2001). The participants in the intervention group received 30 min of tutoring for four days per week, with at least one of the tutoring sessions focusing on the solving of word problems. During the lessons that involved solving word problems the tutors would start the lesson by helping the participants read the problem. Then the tutors would help the participants create a representation of the problem using a picture, a chart, or manipulatives. Finally, the tutor would provide the participants with the program's problem-solving guide and primarily act as a facilitator for the discussion of how to solve the problem. The tutor would only directly teach how to solve a problem if he or she felt that the participants would not be able to complete the problem independently.

Every three weeks, the participants in the intervention group would also receive a performance assessment (Woodward et al., 2001). The day before the participants would be given these assessments, they would spend around 15 min independently practicing a problem that would be similar to those seen on the assessment. After the given practice time, the general education teacher would present a choice of answers to the problem for the class to discuss. Then, after the discussion, the class would be given time to refine their answers if they deemed it necessary. The same process was taken for the actual assessment on the next day, with the exception that students were given 20 min to complete the problem, and that the class discussion and chance to revise answers were given after the assessments were graded.

Results from Woodward et al. (2001) demonstrated that the combination of ad hoc tutoring and whole-class practice on performance assessment tasks was beneficial for participants in the intervention group. These results were taken from the pre- and posttest scores, in addition to ongoing assessments that occurred once every three weeks from October through February during the study's second year (Woodward, Monroe, & Baxter, 2001, p. 43).

The use of ad hoc tutoring is a useful strategy for improving the mathematics word problem solving skills of students with learning disabilities because it has been found that, in the general education classroom, their opportunities to contribute to classroom discussions are minimal (Baxter, Woodward, & Olson, in press). Ad hoc tutoring provides the opportunity for teachers to scaffold instruction, as well as for students to actively participate in discussions (Woodward et al., 2001).

English Language Learners

Finally, how do we help students who are new to the United States and who are considered English Language Learners (ELLs)? For these students, mathematics can be challenging because not only do they have to learn new calculation skills, but they have to learn the English vocabulary terms that correspond with those skills (Freeman & Crawford, 2008). One way to help ELL students become more successful when learning mathematics is to incorporate culturally responsive teaching into our classrooms (Driver & Powell, 2017). The principal idea behind culturally responsive teaching is that we make education accessible to everyone, especially those that have, historically, been an afterthought in the educational system (Hernandez, Morales, & Shroyer, 2013).

For ELLs, culturally responsive teaching has traditionally meant focusing on linguistically responsive teaching, which involves providing aids to help their English language development (e.g., native language, grammatical supports, and vocabulary development) (Echevarria, Short, & Powers, 2006; Goldenberg, 2013). In a 2017 study, Driver and Powell investigated the efficacy that an intervention that integrated linguistically responsive teaching and schema-based instruction (teaching a student to identify a word problem's type) had on ELL students' word-problem solving.

The third-grade participants in this study received three tutoring sessions per week for 10 weeks (Driver & Powell, 2017). The tutoring sessions were broken up into two phases with the focus of the first phase being on the use of the RISE strategy, which taught participants to first read the problem, then illustrate the problem by drawing a visual representation or writing the equation, after which they would solve for the unknown amount, and finally explain what they were solving for. The second phase of the tutoring sessions implemented schema instruction. During the tutoring sessions that involved schema instruction, participants were taught to identify what type of problem they were working with, both known and unknown quantities, and to create the number sentence for the problem. The second phase also included the following strategies for supporting ELL students: stating the objectives of the lesson, encouraging student participants to talk with one another, letting students use their first language, using visual presentations and concrete materials to understand mathematics, incorporating student ideas and experiences, and typing instructional content to the students' lives.

Results from Driver and Powell (2017) showed that there was a significant improvement from pretest to posttest scores for participants that received the intervention. The authors were unable to determine, though, whether the improved scores were attributable to the linguistically responsive teaching or the schema-based instruction portion of the intervention. So, even though further research needs to be done to investigate effective strategies for improving the mathematical abilities of ELL students, this study was able to provide at least one intervention that was found to be helpful for ELL students (Driver, & Powell, 2017).

While the supports provided to ELL students during the second phase of Driver and Powell's (2017) study may or may not have been beneficial, Orosco (2014) that the sole use of these commonly accepted best practices when teaching students that are considered ELLs is most likely not enough to aid in their development of problem-solving skills (Orosco, 2014). So, he wanted to see if the use of a strategy called Dynamic Strategic Math would improve problem-solving abilities in ELL students, and if this strategy would help students to generalize skills to different areas (Orosco, 2014).

Dynamic Strategic Math is based on the idea of dynamic assessment, in which a teacher interacts with students during assessment to not only determine a student's current capabilities, but to determine where the student has the potential to function at as well (Orosco, 2014). Dynamic Strategic Math consists of three steps, the first of which is to use direct instruction to pre-teach the math concepts and vocabulary that will be seen in the word problems used during the lesson (Orosco, 2014). The second step would be to model the problem-solving strategy to be used. Finally, the third step is to group students into pairs, having one act as the teacher to the other student, and observe if and where difficulties occur so that any re-teaching can be focused on those specific difficulties.

The Dynamic Strategic Math framework proved to be beneficial for all of the study's participants, in that each participant was only able to solve word problems with the most basic vocabulary during the baseline phase, but were able to improve to being able to solve problems with either advanced-intermediate, or technical vocabulary during the maintenance phase (Orosco, 2014). So, when it comes to teaching ELL students, one of the most important things that teachers can do is to not only teach potentially unfamiliar vocabulary, but to teach students how to check their own understanding of what they are reading.

Discussion

While reviewing previous research on how to best support students with MD when they are learning how to solve mathematical word problems, there were several findings that stood out.

One of the primary takeaways that should be interpreted from this synthesis is that the successful implementation of cognitive strategies can be an effective way to help students with mathematics difficulties improve their problem solving skills (Montague, & Dietz, 2009). Students' ability to comprehend language, complete cognitive tasks, or improve their metacognitive skills are important aspects of word problem solving, and all of the cognitive strategies discussed in this synthesis worked towards at least one of these goals in some manner: schema-based instruction taught students to understand the various types of word problems (Fuchs et al., 2009), generative strategies taught students how to paraphrase the components of word problems (Moran et al., 2014), both the bar model (Morin, Watson, et al., 2017) and conceptual model-based problem solving (Xin et al., 2011) strategies taught students how to visually represent problems in order to improve comprehension (Morin, Watson, et al., 2017), strategies that dealt with relevant and irrelevant information taught students how to both identify important details as well as ignore extraneous ones (Swanson, Orosco, et al., 2014; Swanson, 2014; Swanson, Lussier, et al., 2013), and the self-explanation strategy taught students how to effectively communicate their work to others (Liu & Xin, 2017).

When choosing which strategy to implement, though, the amount of working memory that a student has available should be kept in mind. Certain strategies, like generative strategies, have been found to be more successful for students with a higher working memory capacity

(Swanson et al., 2014). So when working with students that struggle with working memory a teacher might find that students are more successful with a strategy similar to the one that Swanson, Orosco, et al. (2014) used that focused on the discrimination between relevant and irrelevant information. Students with low working memory were more successful in Swanson, Orosco, et al.'s (2014) study because they were not required to remember as much information.

Another important finding from this synthesis was that students with mathematics difficulties may be more successful if they are allowed to work in smaller groups (Woodward et al., 2001). Not only does working in smaller groups allow the teacher to scaffold instruction more effectively for students (Woodward et al., 2001), but it also encourages more dialogue between students, which in turn helps to promote the learning of struggling students (Kotsopoulos, 2010).

Finally, to ensure that instruction is meeting the needs of all students, teachers should also have strategies readily available that supports students whose first language is not English. To do this, traditional supports like allowing students to use their native language, as well as developing their English vocabulary should be used (Echevarria, Short, & Powers, 2006; Goldenberg, 2013). But, in addition to these traditional supports, strategies like Orosco's (2014) dynamic assessment based strategy should be used as well.

Finding a way to integrate strategies that have proven to be effective over time with more recent research-based interventions is the best way to help struggling students become more successful.

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